CBCS SCHEME

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15MAT31

Third Semester B.E. Degree Examination, July/August 2021
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions.

1 a. Obtain the Fourier series for the function,

$$f(x) = \begin{cases} 1+\dfrac{2x}{\pi} & -\pi \le x \le 0 \\ 1-\dfrac{2x}{\pi} & 0 \le x \le \pi \end{cases}$$

Hence deduce that $\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+.....=\dfrac{\pi^2}{8}$ **(08 Marks)**

b. Find the constant term and first two harmonics in the Fourier series for f(x) given by the following table:

x	0	π/3	2π/3	π	4π/3	5π/3	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(08 Marks)

2 a. Expand $f(x)=\sqrt{1-\cos x}$ in $0 \le x \le 2\pi$ in a Fourier series. Evaluate $\dfrac{1}{1.3}+\dfrac{1}{3.5}+\dfrac{1}{5.7}+.....$ **(08 Marks)**

b. Obtain the Fourier series for f(x) = |x| in $(-\ell, \ell)$ and hence evaluate $\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+.....=\dfrac{\pi^2}{8}$ **(08 Marks)**

3 a. Find the Fourier transform of $f(x)=\begin{cases} 1-|x| & \text{for} \quad |x|\le 1 \\ 0 & \text{for} \quad |x|>1 \end{cases}$ and hence deduce that $\int_0^\infty \dfrac{\sin^2 t}{t^2}\,dt$ **(06 Marks)**

b. Find the Fourier sine transform of $e^{-|x|}$. Hence show that $\int_0^\infty \dfrac{x \sin mx}{1+x^2}dx = \dfrac{\pi}{2}e^{-m}$ where m > 0. **(05 Marks)**

c. Find the z-transform of (i) $(2n-1)^2$ (ii) $\cos\left(\dfrac{n\pi}{2}+\dfrac{\pi}{4}\right)$ **(05 Marks)**

4 a. Find the Fourier transform of $f(n)=\begin{cases} 1 & |x|\le 1 \\ 0 & |x|>a \end{cases}$. Hence deduce $\int_0^\infty \dfrac{\sin ax}{x}dx$. **(06 Marks)**

b. Find the inverse z-transform of $\dfrac{4z^2-2z}{z^3-5z^2+8z-4}$. **(05 Marks)**

c. Solve the differential equation $u_{n+2}+6u_{n+1}+9u_n=2^n$ with $u_0 = u_1 = 0$ using z-transform method. **(05 Marks)**

5 a. Find the coefficient of correlation and the two lines of regression for the following data:

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

(06 Marks)

b. Fit a curve of the form $y = ae^{bx}$ to the following data:

x	77	100	185	239	285
y	2.4	3.4	7.0	11.1	19.6

(05 Marks)

c. Use Regula Falsi method, find the root of the equation $x^2 - \log_e x - 12 = 0$. (05 Marks)

6 a. The two regression equations of the variables x and y are x = 19.13 – 0.87y and y = 11.64 – 0.5x. Find:
(i) Means of x
(ii) Means of y
(iii) The correlation coefficient (06 Marks)

b. Fit a parabola $y = a + bx + cx^2$ to the following data:

x	–3	–2	–1	0	1	2	3
y	4.63	2.11	0.67	0.09	0.63	2.15	4.58

(05 Marks)

c. Use Newton-Raphson method to find the real root of $3x = \cos x + 1$, take $x_0 = 0.6$ perform 2 iterations. (05 Marks)

7 a. Find the cubic polynomial by using Newton forward interpolating formula which takes the following values.

x	0	1	2	3
y	1	2	1	10

(06 Marks)

b. Apply Lagrange's formula inversely to obtain a root of the equation f(x) = 0 given that f(30) = –30, f(34) = –13, f(38) = 3, f(42) = 18. (05 Marks)

c. Use Weddle's rule to evaluate $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ dividing the interval $\left[0, \frac{\pi}{2}\right]$ into six equal parts. (05 Marks)

8 a. A survey conducted in a slum locality reveals the following interpolating information as classified below:

Income/day in rupees : x	Under 10	10-20	20-30	30-40	40-50
Number of persons : y	20	45	115	210	115

Estimate the probable number of persons in the income group 20 to 25. (06 Marks)

b. Using Newton divided difference formula fit an interpolating polynomial for the following data:

x	0	1	4	5
f(x)	8	11	68	123

(05 Marks)

c. Using Simpson's 1/3rd rule evaluate $\int_0^1 \frac{x^2}{1+x^3} dx$ taking four equal strips. (05 Marks)

9 a. Find the extremal of the functional $I = \int_0^{\pi/2} (y^2 - y'^2 - 2y\sin x)dx$ under the conditions $y(0) = y\left(\frac{\pi}{2}\right) = 0$. **(06 Marks)**

b. If $\vec{F} = x^2 i + xyj$ evaluate $\int_C \vec{F}.d\vec{r}$ from (0, 0) to (1, 1) along

(i) the line $y = x$ (ii) the parabola $y = \sqrt{x}$ **(05 Marks)**

c. Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about the x-axis gives a minimum surface area. **(05 Marks)**

10 a. Verify Green's theorem in a plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$. **(06 Marks)**

b. Using divergence theorem evaluate $\int \vec{A} \cdot \hat{n}\, ds$ where $\vec{A} = x^3 i + y^3 j + z^3 k$ and s is the surface of the surface $x^2 + y^2 + z^2 = a^2$. **(05 Marks)**

c. Find the geodesics on a surface given that the arc length on the surface is $s = \int_{x_1}^{x_2} \sqrt{x(1 + y'^2)}dx$. **(05 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | | **15EC36**

Third Semester B.E. Degree Examination, Aug./Sept.2020
Engineering Electromagnetics

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Define Electric Field Intensity , $\vec{E}$. Find $\vec{E}$ at $(2, \frac{\pi}{2}, \frac{\pi}{6})$ due to a point charge located at origin. Let Q = 40nC. **(04 Marks)**

b. Point charges of 120nC are located at A (0, 0, 1) and B(0, 0, -1) in free space. Find $\vec{E}$ at P(x, 0, 0). Also find the maximum value of $\vec{E}$. **(06 Marks)**

c. Uniform line charges of 120 nC/m each lie along the entire extent of the three co-ordinate axes. Assuming free space conditions, find $\vec{E}$ at P(-3, 2, -1)m. **(06 Marks)**

OR

2 a. Derive an expression for electric field intensity at a point in cylindrical coordinate system due to an infinite line charge distribution on Z - axis. **(06 Marks)**

b. A point charge Q_1 = 10 μC is located at P_1(1, 2, 3)m in free space while Q_2 = -5μC is at P_2(1, 2, 10)m. i) Find vector force exerted on Q_2 by Q_1 ii) Also, find the co-ordinates of P_3 at which a point charge Q_3 experiences no force. **(07 Marks)**

c. Find the total electric flux crossing an infinite plane at y = 0 due to the following charge distributions : • a point charge, 30nC located at (1, 2, 3).
• Two line charge distributions of 10nC/m each located in x = 0 plane at y = ± 2m extending over a length of 4m. **(03 Marks)**

Module-2

3 a. Define 'Divergence of a Vector' and 'Gradient of a Scalar'. **(04 Marks)**

b. Derive the point form of Gauss's law. **(06 Marks)**

c. Give the flux density , $\vec{D} = \frac{5\sin\theta\cos\phi}{r}\hat{a}_r$, c/m^2. Find • Volume charge density
- Total charge contained in the region, r < 2m.
- Total electric flux leaving the surface, r = 2m. **(06 Marks)**

OR

4 a. The value of $\vec{E}$ at P(ρ = 2 , ϕ = 40^0, Z = 3) is given by $\vec{E} = 100\,\hat{a}_\rho - 200\,\hat{a}_\phi + 300\,\hat{a}_z$, V/m. Determine the incremental work required to move a 20μC charge a distance of 6μm in the direction of : i) $\hat{a}_\rho$ ii) $\vec{E}$ iii) $\vec{G} = \hat{a}_\rho + 3\,\hat{a}_\phi - 2\,\hat{a}_z$. **(06 Marks)**

b. State and explain continuity equation of current. **(05 Marks)**

c. Given the potential field $V = 2x^2y - 80$, and a point, P(2, 3, -4) in free space, find at 'P'.
i) V ii) $\vec{E}$ iii) $\frac{dV}{dN}$ iv) $\hat{a}_N$.
Where $\hat{a}_N$ is the unit vector normal to equipotential surface? **(05 Marks)**

Module-3

5 a. Conducting plates at Z = 2cm and Z = 8cm are held at potentials of -3V and 9V respectively. The region between the plates is filled with a perfect dielectric of $\epsilon = 5\epsilon_0$.
Find V, $\vec{E}$ and $\vec{D}$ in the region between the plates. **(06 Marks)**

b. Let $V = \frac{\cos 2\phi}{\rho}$ volts in free space. Find volume charge density at $P(5, 60^0, 1)$ using Poisson's equation. **(05 Marks)**

c. State the following : i) Uniqueness theorem ii) Ampere's law iii) Stoke's theorem. **(05 Marks)**

OR

6 a. Explain Scalar and Vector magnetic potentials. **(05 Marks)**

b. Verify Stoke's theorem for $\vec{H} = 2r\cos\theta\ \hat{a}_r + r\ \hat{a}_\phi$ for the path defined by $0 \le r \le 1$ and $0 \le \theta \le 90^0$. **(06 Marks)**

c. The magnetic field intensity is given by $\vec{H} = 0.1\ y^3\ \hat{a}_x + 0.4\ x\ \hat{a}_z$, A/m. Determine the current flow through the path $P_1(5, 4, 1)$ to $P_2(5, 6, 1)$ to $P_3(0, 6, 1)$ to $(0, 4, 1)$. Also find current density, $\vec{J}$. **(05 Marks)**

Module-4

7 a. Obtain an expression for magnetic force between differential current elements. **(05 Marks)**

b. A point charge, Q = 18nC has a velocity of 5×10^6 m/s in the direction $\hat{a} = 0.6\ \hat{a}_x + 0.75\ \hat{a}_y + 0.3\ \hat{a}_z$. Calculate the magnitude of the force exerted on the charge by the field $\vec{B} = -3\ \hat{a}_x + 4\ \hat{a}_y + 6\ \hat{a}_z$, mT. **(05 Marks)**

c. Three infinitely long parallel filaments each carry 50A in the $\hat{a}_z$ direction. If the filament lie in the plane, x = 0 with a 2cm spacing between wires, find the vector fore per meter on each filament. **(06 Marks)**

OR

8 a. Obtain the boundary conditions at the interface between two magnetic materials. **(05 Marks)**

b. Find Magnetization in magnetic material where
i) $\mu = 1.8 \times 10^{-5}$ H/m and H = 120 A/m ii) B = 300μT and $X_m = 15$. **(05 Marks)**

c. Explain briefly the following as applicable to magnetic materials :
i) Magnetization ii) Permeability iii) Potential energy. **(06 Marks)**

Module-5

9 a. Write Maxwell's equations in integral form and word statement form for free space. **(06 Marks)**

b. In a certain dielectric medium, $\epsilon_r = 5$, $\sigma = 0$ and displacement current density $\vec{J}_d = 20\cos(1.5 \times 10^8 t - bx)\ \hat{a}_y$, μA/m². Determine electric flux density and electric field intensity. **(06 Marks)**

c. A radial magnetic field $\vec{H} = \frac{2.239 \times 10^6}{r}\cos\phi\ \hat{a}_r$, a/m exists in free space. Find the magnetic flux, ϕ crossing the surface defined by $-\frac{\pi}{4} \le \phi \le \frac{\pi}{4}$, $0 \le z \le 1$, m. **(04 Marks)**

OR

10 a. Discuss the wave propagation of a uniform plane wave in a good conducting medium. **(06 Marks)**

b. Derive the relation between $\vec{E}$ and $\vec{H}$ for a perfect dielectric medium. **(05 Marks)**

c. Determine the skin depth for copper with conductivity of 58×10^6, S/m at a frequency, 10 MHz. Also find α, β and V_p. **(05 Marks)**

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CBCS SCHEME

USN

15MATDIP31

Third Semester B.E. Degree Examination, July/August 2021
Additional Mathematics - I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Express $\frac{(3+i)(1-3i)}{(2+i)}$ in the form x + iy. **(06 Marks)**

b. Find the modulus and amplitude of the complex number $1 + \cos\alpha + i\sin\alpha$. **(05 Marks)**

c. If $\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = \hat{i} - 2\hat{j} + 2\hat{k}$, then find $\vec{a} \times (\vec{b} \times \vec{c})$. **(05 Marks)**

2 a. Prove that $\left[\frac{1+\cos\theta + i\sin\theta}{1+\cos\theta - i\sin\theta}\right]^n = \cos n\theta + i\sin n\theta$. **(06 Marks)**

b. Find the cube root of $1 + i\sqrt{3}$. **(05 Marks)**

c. Show that the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$ are coplanar. **(05 Marks)**

3 a. Find the n^{th} derivative of $e^{ax}\sin(bx + c)$. **(06 Marks)**

b. With usual notations prove that $\tan\phi = r \cdot d\theta/dr$. **(05 Marks)**

c. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then show that $x\frac{\partial u}{\partial x} + y\,\partial u/\partial y = \sin 2u$. **(05 Marks)**

4 a. Find the n^{th} derivative of $\frac{x}{(x-2)(x-3)}$. **(06 Marks)**

b. Find the angle between the curves $r = a(1+\cos\theta)$ and $r = b(1 - \cos\theta)$. **(05 Marks)**

c. Given $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. **(05 Marks)**

5 a. Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x \, dx$. **(06 Marks)**

b. Evaluate $\int_0^{\pi/16} \cos^5(8x)\sin^6(16x)\,dx$. **(05 Marks)**

c. Evaluate $\int_1^2\int_1^3 x y^2 \,dx\,dy$. **(05 Marks)**

6 a. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$. **(06 Marks)**

b. Evaluate $\int_0^{\pi} \frac{\sin^4\theta}{(1+\cos\theta)^2}\,d\theta$. **(05 Marks)**

c. Evaluate $\int_{-3}^{3}\int_0^1\int_1^2 (x+y+z)\,dx\,dy\,dz$. **(05 Marks)**

7 a. Find velocity and acceleration of a particle moving along the curve $\vec{r} = e^{-2t}\,\hat{i} + 2\cos 5t\,\hat{j} + 5\sin t\,\hat{k}$ at anytime t. Find their magnitudes at t = 0. **(06 Marks)**

b. If $\phi = x^3 + y^3 + z^3 - 3xyz$ find $\nabla\phi$ at (1, -1, 2). **(05 Marks)**

c. Show that $\vec{F} = (x + 3y)\,\hat{i} + (y - 3z)\,\hat{j} + (x - 2z)\,\hat{k}$ is Solenoidal. **(05 Marks)**

8 a. Find the unit tangent vector of the space curve $\vec{r} = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}$. **(06 Marks)**

b. If $\vec{F} = x^2y\,\hat{i} + yz^2\,\hat{j} + zx^2\,\hat{k}$, then find div (curl $\vec{F}$). **(05 Marks)**

c. Find the constants a, b and c such that the vector $\vec{F} = (x + y + az)\,\hat{i} + (x + cy + 2z)\,\hat{j} + (bx + 2y - z)\,\hat{k}$ is irrotational. **(05 Marks)**

9 a. Solve $\frac{dy}{dx} = 1 + y/x + \left(y/x\right)^2$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + y\cot x = \sin x$. **(05 Marks)**

c. Solve $\frac{dy}{dx} = \frac{x^2 - 2xy}{x^2 - \sin y}$. **(05 Marks)**

10 a. Solve $(2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0$. **(06 Marks)**

b. Solve $(1 + xy)y\,dx + (1 - xy)\,x\,dy = 0$. **(05 Marks)**

c. Solve $x\frac{dy}{dx} + y = x^3y^6$. **(05 Marks)**

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